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# Penrose limits of orbifolds and orientifolds

**Emmanuel Floratos**[ab] **and Alex Kehagias**[ac]

[a] *Institute of Nuclear Physics, N.C.R.P.S. Democritos GR-15310, Athens, Greece*
[b] *Nuclear and Particle Physics Sector, Univ. of Athens GR-15771 Athens, Greece*
[c] *Physics Department, NTUA GR-15773 Zografou, Athens, Greece*
*E-mail:* manolis@mail.democritos.gr, kehagias@mail.cern.ch

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Abstract: We study the Penrose limit of various $AdS_p \times S^q$ orbifolds. The limiting spaces are waves with parallel rays and singular wave fronts. In particular, we consider the orbifolds $AdS_3 \times S^3/\Gamma$, $AdS_5 \times S^5/\Gamma$ and $AdS_{4,7} \times S^{7,4}/\Gamma$ where $\Gamma$ acts on the sphere and/or the AdS factor. In the pp-wave limit the wave fronts are the orbifolds $\mathbb{C}^2/\Gamma$, $\mathbb{C}^4/\Gamma$ and $\mathbb{R} \times \mathbb{C}^4/\Gamma$, respectively. When desingularization is possible, we get asymptotically locally pp-wave backgrounds (ALpp). The Penrose limit of the $AdS_5 \times \mathbb{RP}^5$ orientifold as well as the pp-wave limit of D3-branes near singularities in F-theory are also presented. Finally, we give the embedding of D-dimensional pp-waves in flat $M^{2,D}$ space.

Keywords: AdS-CFT and dS-CFT Correspondence, D-branes, Superstring Vacua.

## Contents





## 1. Introduction

Plane wave backgrounds with parallel rays (pp-waves) are known to be exact string solutions [1, 2]. Such backgrounds have reconsidered recently [3, 4, 5] since they share the similar properties with flat and $AdS_p \times S^q$ spaces (e.g. maximal supersymmetry). The pp-wave background we will discuss can be considered as the Penrose limit [7] of AdS spaces [3, 4, 5, 8, 9]. The limit is particular and amounts in boosting around maximal cycles in $S^q$ and then flatten the resulting background by taking the standard large radius limit of both $AdS_p$ and $S^q$. This limit has also a well defined action on the superalgebras of $AdS_p \times S^q$. The latter are members of infinite sequence of superalgebras usually denoted by $\mathrm{OSp}(N|M)$ and $\mathrm{SU}(N|M)$ (There is also the isolated $F(4)$ superalgebra corresponding to $AdS_6$) [10]. The superalgebras of the limiting pp-wave backgrounds of $AdS_p \times S^q$ can be constructed by contraction of the of $\mathrm{OSp}(N|M)$ or $\mathrm{SU}(N|M)$. An explicit demonstration of this has been carried out in [11] for the $\mathrm{SU}(2,2|4)$ superalgebra.

The same limits can also be considered in 11D $AdS_{4,7} \times S^{7,4}$ M-theory backgrounds. Here, the resulting pp-wave has been constructed in the past [6] and has recently be recovered [3]. An interesting aspects of all these constructions is that the Penrose limit preserves all supersymmetries of the original background. Thus, the resulting pp-waves are maximally supersymmetric. The fact that maximally supersymmetric space-times have played a central role in understanding string and M-theory explains in part the recent interest in these pp-wave backgrounds. Additional support for studying them, came from the recent proposal of Berestein, Maldacena and Nastase (BMN) [9] that the string spectrum on a pp-wave background arises from the large-$N$ limit of $\mathcal{N}=4$ SYM theory in 4d. This has been demonstrated by summing a subset of planar diagrams and it is a very interesting extension of the original AdS/CFT correspondence as it involves massive string modes. The large $N$ limit that has been employed is not just large 't Hooft coupling $g_{YM}^2 N \to \infty$ but also fixed $g_{eff}^2 = g_{YM}^2 N/J^2$ ($J$ is a global charge). Then, correlation functions for operators of scaling dimension $\Delta$ are calculated in large 't Hooft coupling, fixed $g_{eff}$ and finite $\Delta - J$.

Here, we consider various orbifolds of $AdS_p \times S^q$ and their pp-wave limit. As these orbifolds have less supersymmetry, the corresponding pp-waves will also have less supersymmerties provided that the singularities are not washed out in the limit. The spectrum on these backgrounds should then arise from the corresponding SCFTs in the BMN limit. The same can be done for AdS orientifolds and we present the pp-wave limits of the near horizon of D3-branes on O3-plane as well as in F-theory (at constant coupling) [12, 13, 14].

This paper is organized as follows. In section 2, we describe the Penrose limits of $AdS_p \times S^q$ backgrounds which occur in string and M-theory. In section 3 we construct various orbifolds of the backgrounds presented in the previous section and we discuss their possible desingularizations. In section 4, we present the pp-wave limit of $AdS_5 \times S^5$ orientifold. Finally, in appendix A we give the embedding of the D-dimensional pp-wave in flat $M^{2,D-1}$ space, while in appendix B we describe the ALE space $\mathbb{C}^3/\mathbb{Z}_3$ we used as an example in section 3.

## 2. Penrose limit of $AdS_p \times S^q$

AdS spaces arise in many instances in string and M-theory. In fact, supersymmetric $AdS_p$ backgrounds are possible, according to [10], for $p = 2, 3, 4, 5, 6, 7$. They appear as vacuum of 10 and 11 dimensional supergravity as well as the near horizon limit of D-brane and M-brane configurations (see [17] for a review). Keeping the discussion as general as possible, let us recall the Penrose limit of $AdS_p \times S^q$, consider first in [4]. The metric of $AdS_p \times S^q$ is

$$ds^2 = R_A^2 \left(d\rho^2 + \sinh^2 \rho d\Omega_{p-2}^2 - \cosh^2 \rho dt^2\right) + R_S^2 \left(d\theta^2 + \sin^2 \theta d\Omega_{q-2}^2 + \cos^2 \theta d\psi^2\right), \quad (2.1)$$

where $R_A, R_S$ are the radius of $AdS_p$ and $S^q$, respectively. We will consider the limiting geometry seen by a fast moving observer in the $\psi$ direction at $\rho = 0, \theta = 0$. For this, we introduce the coordinates

$$x^+ = \frac{t + \alpha\psi}{2}, \qquad x^- = R_A^2 \left(\frac{t - \alpha\psi}{2}\right), \qquad x = R_A \rho, \qquad y = R_S \theta, \qquad \alpha = \frac{R_S}{R_A} \quad (2.2)$$



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in the metric (2.1) and then take the limit $R_{A,S} \to \infty$, keeping $\alpha$ finite. The resulting space-time is non singular and its metric is

$$ds_P^2 = -4dx^+dx^- - (x^2 + \alpha^{-2}y^2)dx^{+2} + dx^2 + x^2 d\Omega_{p-2}^2 + dy^2 + y^2 d\Omega_{q-2}^2 . \qquad (2.3)$$

Next, we will examine each possible AdS-geometries appear in string and M-theory (except the $AdS_2$ and $AdS_6$ cases).

### 2.1 $AdS_3 \times S^3$

This background appears in the near horizon geometry of the D1/D5 system [18]. The 10D geometry is $AdS_3 \times S^3 \times M^4$ and the D5-branes are wrapped on $M^4$. In order that supersymmetry is preserved, $M^4$ is either $T^4$ or $K^3$ for 8 or 4 surviving supersymetries. Omitting the irrelevant for the discussion $M^4$ factor, the bosonic symmetry of this background is $\mathrm{SO}(2,2)\times\mathrm{SO}(4)$ which is the bosonic part of the 2D SCFT living in the boundary of $AdS_3$ [19]. The metric is

$$ds^2 = R_A^2 \left(d\rho^2 + \sinh^2\rho d\phi^2 - \cosh^2\rho dt^2\right) + R_S^2 \left(d\theta^2 + \sin^2\theta d\chi^2 + \cos^2\theta d\psi^2\right), \qquad (2.4)$$

with $R_A = R_S$. Defining the coordinates as in (2.2) and taking the $R_A \to \infty$ limit, we end up with the pp-wave

$$\begin{aligned} ds_P^2 &= -4dx^+dx^- - \mu^2(x^2+y^2)dx^{+2} + dx^2 + x^2 d\phi^2 + dy^2 + y^2 d\chi^2 \\ H_{+34} &= H_{+56} = 2\mu , \end{aligned} \qquad (2.5)$$

where $H$ is the RR three-form of the D1/D5 system. It is evident that the pp-wave front $x^+$ =const. is flat 4D euclidean space.

### 2.2 $AdS_5 \times S^5$

The $AdS_5 \times S^5$ background is by far the most celebrated one. It is realized as the near horizon limit of $N$ coincident D3-branes and it is maximally supersymmetric (32 supersymmetries). $AdS_5 \times S^5$ with $N$ units of five-form flux is conjectured to be the supergravity dual of $\mathrm{SU}(N)$ $\mathcal{N}=4$ gauge theory at large-$N$ [19]. The bosonic symmetry of the background is $\mathrm{SO}(4,2)\times\mathrm{SO}(6)$. The $\mathrm{SO}(4,2)$ part is realized as the conformal symmetry of the $\mathrm{SU}(N)$ $\mathcal{N}$ gauge theory while $\mathrm{SO}(6)$ is the R-symmetry. Starting form the $AdS_5 \times S^5$ metric

$$ds^2 = R^2 \left(d\rho^2 + \sinh^2\rho d\Omega_3^2 - \cosh^2\rho dt^2 + d\theta^2 + \sin^2\theta d\Omega_3^2 + \cos^2\theta d\psi^2\right), \qquad (2.6)$$

the geometry seen by a fast moving observer in the $\psi$ direction sitting at $\rho=0,\ \theta=0$ can be obtained, according to eq. (2.2), by introducing the coordinates

$$x^+ = \frac{t+\psi}{2}, \qquad x^- = R^2\left(\frac{t-\psi}{2}\right), \qquad x = R\rho, \qquad y = R\theta, \qquad (2.7)$$

and then take the limit $R \to \infty$. The resulting non-singular space-time metric and the surviving RR self-dual five-form are[1]

$$\begin{aligned} ds^2 &= -4dx^+dx^- - \mu^2 \vec{r}^{\,2} dx^{+2} + d\vec{r}^{\,2} \,, \\ F_{+1234} &= F_{+5678} = 4\mu \end{aligned} \tag{2.8}$$

where $\vec{r}^{\,2} = \vec{x}^{\,2} + \vec{y}^{\,2}$. Clearly, the wave front is flat 8D euclidean space. The type IIB superstring in the above background has been argued to be exactly solvable, described by free massive fields [20, 21, 22].

### 2.3 $AdS_4 \times S^7$

This geometry appears in the near horizon of $N$ coincident M2-branes in M-theory. It is the supergravity dual of $\mathcal{N} = 8$ 3D SCFT living on the M2 worldvolume. The bosonic symmetry of the supergravity background is SO(3, 2) × SO(8). As usual, the SO(3, 2) is identified with the conformal symmetry of the boundary SCFT while the SO(8) part is the R-symmetry group. Starting form the $AdS_4 \times S^7$ metric

$$ds^2 = R_A^2 \left(d\rho^2 + \sinh^2\rho\, d\Omega_2^2 - \cosh^2\rho\, dt^2\right) + R_S^2 \left(d\theta^2 + \sin^2\theta\, d\Omega_5^2 + \cos^2\theta\, d\psi^2\right) , \tag{2.9}$$

where $R_S = 2R_A$, the Penrose limit is obtained by defining

$$x^+ = \frac{1}{2}(t + 2\psi) \,, \qquad x^- = \frac{R_A^2}{2}(t - 2\psi) \,, \qquad x = R_A\rho \,, \qquad y = 2R_A\theta \,, \tag{2.10}$$

and letting $R_A \to \infty$. The resulting space-time metric and 4-form are

$$\begin{aligned} ds_P^2 &= -4dx^+dx^- - \mu^2\left(x^2 + \frac{1}{4}y^2\right) dx^{+2} + dx^2 + x^2 d\Omega_2^2 + dy^2 + y^2 d\Omega_5^2 \,, \\ F_4 &= 3\mu\, dx^+ \wedge dx^1 \wedge dx^2 \wedge dx^3 \end{aligned} \tag{2.11}$$

so that the near horizon of M2-branes has been turned into pp-waves with flat wave fronts.

### 2.4 $AdS_7 \times S^4$

Similarly to the above, this geometry is realized in the near horizon limit of $N$ coincident M5 branes in M-theory. They describe the supergravity duals of large-$N$ 6D SCFT. The bosonic symmetry of this background is SO(6, 2) × SO(5) which is the bosonic part of the 6D SCFT. The metric of $AdS_7 \times S^4$ is

$$ds^2 = R_A^2 \left(d\rho^2 + \sinh^2\rho\, d\Omega_5^2 - \cosh^2\rho\, dt^2\right) + R_S^2 \left(d\theta^2 + \sin^2\theta\, d\Omega_2^2 + \cos^2\theta\, d\psi^2\right) . \tag{2.12}$$

Here, $R_A = 2R_S$ and the limit $R_A \to \infty$ after the transformation

$$x^+ = \frac{1}{2}\left(t + \frac{\psi}{2}\right) , \qquad x^- = \frac{R_A^2}{2}\left(t - \frac{\psi}{2}\right) , \qquad x = R_A\rho \,, \qquad y = \frac{R_A}{2}\theta \,, \tag{2.13}$$

[1] This is exactly the metric on the group manifold of the 10D Heisenberg group [23]. It is a generalization of the Nappi-Witten background [24] with RR fields instead of NS/NS antisymmetric 2-form. It can also be considered as a particular contruction of a WZW model [25].



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leads to the pp-wave background and 4-form

$$
\begin{aligned}
ds_P^2 &= -4dx^+dx^- - (x^2+4y^2)dx^{+2} + dx^2 + x^2 d\Omega_5^2 + dy^2 + y^2 d\Omega_2^2\,,\\
F_4 &= 6\mu\, dx^+ \wedge dy^1 \wedge dy^2 \wedge dy^3\,.
\end{aligned}
\tag{2.14}
$$

In fact, the pp-wave limits of both $AdS_4 \times S^7$ and $AdS^7 \times S^4$ in eqs. (2.11) and (2.14), respectively, are the same (as can be seen by the transformation $x \leftrightarrow y$, $x^+ \to x^+/2$) and the two maximally supersymmetric backgrounds (2.9) and (2.12) have the unique maximally supersymmetric pp-wave background (2.11) (or (2.14)) found in [6].

## 3. Penrose limits of $AdS_p \times S^q$ orbifolds

Branes can be put at conifold [26, 27, 28] or orbifold singularities [29, 30, 31]. The near horizon limit of such configurations lead to either singular or non-singular spaces. Here we will examine orbifolds of $AdS_p \times S^q$ and we will find their pp-wave limit. The corresponding limit at the conifold has been studied in [32, 33, 34]. The type of orbifold theories we will consider are such that they lead to singularities on the wave front of the pp-wave. Some of these singularities may be resolved to a smooth Ricci flat space (leading not to flat wave fronts but rather to Ricci flat ones) and some not. We will discuss each case of AdS background separately.

### 3.1 $AdS_3 \times S^3$ orbifolds

This background may be embedded in 8D space-time $M^{2,6}$ with metric (in an obvious complex notation)

$$
ds_8^2 = -|dZ_0|^2 + |dZ_1| + |dZ_2|^2 + |dZ_3|^2\,, \tag{3.1}
$$

as the hypersurface

$$
|Z_0|^2 - |Z_1|^2 = R^2\,, \qquad |Z_2|^2 + |Z_3|^2 = R^2\,. \tag{3.2}
$$

The parametrization

$$
Z_0 = R\cosh\rho e^{it}\,, \qquad Z_1 = R\sinh\rho e^{i\phi}\,, \qquad Z_2 = R\cos\theta e^{i\psi} \qquad Z_3 = R\sin\theta e^{i\chi}\,, \tag{3.3}
$$

leads directly to the metric (2.4). Many orbifolds of the hypersurface (3.2) may be taken. For example, we may consider the $\mathbb{Z}_k$ action

$$
Z_3 \to e^{2\pi i/k} Z_3\,,
$$

which has fixed points the cycle $|Z_2|^2 = R^2$. The pp-wave limit is given again by eq. (2.5). The only difference is the periodicity of $\chi$ which instead being $2\pi$ is now $2\pi/k$. As a result, the wave front has been turned from $\mathbb{C}^2$ in the $AdS_3 \times S^3$, to $\mathbb{C} \times \mathbb{C}/\mathbb{Z}_k$ in the $AdS_3 \times S^3/\mathbb{Z}_k$ case. There exist a conical singularity (at $y = 0$ in (2.5)) which, however, cannot be resolved. An example, where the singularity can be resolved is provided by the orbifold action

$$
\mathbb{Z}_k: \quad Z_3 \to e^{2\pi i/k} Z_3\,, \qquad Z_1 \to e^{-2\pi i/k} Z_1\,. \tag{3.4}
$$

Here, the pp-wave limit is again given by the metric in (2.5), where now the periodicities of both $\phi, \chi$ are $2\pi/k$. (Orbifolds of the AdS factor has been considered in AdS/CFT correspondence in [35, 36]). In fact, the action (3.4) identifies $(\phi, \chi) \equiv (\phi + 2\pi/k, \chi + 2\pi/k)$. Thus, the wave front is $\mathbb{C}^2/Z_k$, which can be resolved by an ALE space.

Replacing $\mathbb{C}^2/\Gamma$ by an ALE space is not the end of the story. We should also make sure that supergravity equations are satisfied. We will do this below for the general case by looking for background with metric and three-form RR field of the form

$$\begin{aligned} ds^2 &= -4dx^+dx^- - S(x^i)dx^{+2} + g_{ij}dx^i dx^i \,, \qquad i,j = 1,\ldots,4\,, \\ H_{+ij} &= \frac{1}{2}\epsilon_{ijmn}H_+{}^{mn}\,, \end{aligned} \tag{3.5}$$

where $g_{ij}$ is the metric of a 4D-dimensional wave-front space $K^4$. The Ricci tensor is

$$R_{++} = \frac{1}{2}\nabla^i\nabla_i S\,, \qquad R_{ij} = R_{ij}(g)\,, \tag{3.6}$$

so that the Einstein equations

$$R_{MN} = \frac{1}{4}\left(H_{MKL}H_N{}^{KL} - \frac{1}{12}H^2 g_{MN}\right), \tag{3.7}$$

reduce to

$$\begin{aligned} \frac{1}{2}\nabla^i\nabla_i S &= \frac{1}{4}H_{+mn}H_+{}^{mn}\,, \\ R_{ij}(g) &= 0\,. \end{aligned} \tag{3.8}$$

In the Penrose limit of $(AdS_3 \times S^3)/\mathbb{Z}_k$ with $\mathbb{Z}_k$ as in eq. (3.4), $H_{MNP}$ is given by eq. (2.5) so that

$$ds^2 = -4dx^+dx^- - \mu^2 S(x)dx^{+2} + g_{ij}dx^i dx^j\,, \tag{3.9}$$

where $g_{ij}$ is the metric of the ALE space. $S$ is determined by the equation

$$\nabla^i\nabla_i S = 8\,. \tag{3.10}$$

In the case of an $AdS_3 \times S^3/\mathbb{Z}_2$ orbifold, the singular $\mathbb{C}/\mathbb{Z}_2$ can be replaced by the Eguchi-Hanson gravitational instanton with metric

$$ds^2_{EH} = \frac{dr^2}{1 - a^4/r^4} + r^2(\sigma_1^2 + \sigma_2^2) + r^2\left(1 - \frac{a^4}{r^4}\right)\sigma_3^2\,. \tag{3.11}$$

The solution to eq. (3.10) for the EH metric then leads to the pp-wave limit of $(AdS_3 \times S^3)/\mathbb{Z}_2$

$$ds^2 = -4dx^+dx^- - \mu^2\left(r^2 + \frac{a^2}{2}\ln\left(\frac{r^2 - a^2}{r^2 + a^2}\right)\right)dx^{+2} + ds^2_{EH}\,. \tag{3.12}$$

Clearly, at $r \to \infty$, the wave fronts of (3.12) become $\mathbb{C}^2/\mathbb{Z}_k$ as it should. In the case of $\mathbb{Z}_k \subset \mathrm{SU}(2)\,, (k > 2)$, $\mathbb{C}^2/\mathbb{Z}_k$ should be replaced by the Gibbons-Hawking multi-center gravitational instanton with metric

$$ds^2_{GH} = V(d\tau + \vec{\omega}\cdot d\vec{x})^2 + V^{-1}d\vec{x}^2\,, \tag{3.13}$$



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where

$$V^{-1}=\sum_{i=1}^{k}\frac{1}{|\vec{x}-\vec{x}_i|}\,,\qquad \vec{\nabla}\times\vec{\omega}=\vec{\nabla}V^{-1}\,. \tag{3.14}$$

Then, the solution of eq. (3.10) will formally be given as

$$S=-\frac{1}{\pi}\sum_{i=1}^{k}\int\frac{d^3x'}{|\vec{x}-\vec{x}'||\vec{x}'-\vec{x}_i|}\,. \tag{3.15}$$

It is clear, that other orbifolds $(AdS_3\times S^3)/\Gamma$ may be considered, where $\Gamma\subset \mathrm{SU}(2)$ in order to preserve supersymmetry (such orbifolds preserve half the supersymmetry $AdS_3\times S^3$ preserves). Their pp-wave limits have then the singular wave fronts $\mathbb{C}^2/\Gamma$ which can be made smooth by replacing them (after appropriate modification of the metric as shown above) by an ALE space. Such spaces may be called ALpp spaces as they are asymptotically (in the wave-front sense) locally pp-waves.

### 3.2 Orbifolds of $AdS_5\times S^5$

Similarly to $AdS_3\times S^3$, $AdS_5\times S^5$ can be embedded in 12D flat space-time $M^{2,10}$ with metric

$$ds_{12}^2=-dX_0^2+dX_1^2+\ldots+dX_{10}^2-dX_{11}^2\,, \tag{3.16}$$

as the hypersurface

$$X_0^2-X_1^2-X_2^2-X_3^2-X_4^2+X_{11}^2=R^2\,, \tag{3.17}$$

$$|Z_1|^2+|Z_2|^2+|Z_3|^2=R^2\,, \tag{3.18}$$

where $Z_1=X_5+iX_6,\ Z_2=X_7+iX_8,\ Z_3=X_9+iX_{10}$. The parametrization of (3.17)

$$\begin{aligned} X_0&=R\cosh\rho\cos t\,, & X_{11}&=R\cosh\rho\,\sin t\\ X_a&=R\sinh\rho\Omega_a\,, & &\left(a=1,\ldots,4\,,\qquad\sum_a\Omega_a^2=1\right), \end{aligned} \tag{3.19}$$

with $0\le\rho<\infty$ and $0\le t<2\pi$ covers the whole of the (3.17) hyperboloid. Together with the angular coordinates $\Omega_a$, which parametrize a unit $S^3$, are the global coordinates of the $AdS_5$. Similarly, the $S^5$ can be parametrized as

$$\begin{aligned} Z_1&=R\cos\theta\,e^{i\psi}\,, & Z_2&=R\sin\theta\,U_1\,,\\ Z_3&=R\sin\theta\,U_2\,, & |U_1|^2&+|U_2|^2=1\,, \end{aligned} \tag{3.20}$$

where $0\le\theta<\pi/2$, $0\le\psi<2\pi$ and the complex $U_{1,2}$ form a unit $S^3$. We may act now with a discrete subgroup $\Gamma$ of the isometry group $\mathrm{SO}(6)\sim\mathrm{SU}(4)$ of $S^5$. There are two distinct cases which preserve supersymmetry [31, 37]. The first one is when $\Gamma\subset\mathrm{SU}(3)\subset\mathrm{SU}(4)$ and leads to $\mathcal{N}=1$ supersymmetric $\mathrm{SU}(N)$ gauge theory. The second case is when $\Gamma\subset\mathrm{SU}(2)\subset\mathrm{SU}(4)$ with $\mathcal{N}=2$ supersymmetry. Both cases have the same Penrose limit.

For concreteness, we will consider a $\Gamma \subset \mathrm{SU}(2)$ acting on the coordinates $Z_2, Z_3$. The $\Gamma$ action fixes the $S^1$ in $S^5$

$$|Z_1|^2 = 1\,, \qquad Z_2 = Z_3 = 0\,, \tag{3.21}$$

and thus $S^5/\Gamma$ is a singular orbifold. In the parametrization (3.20), $\Gamma$ acts freely on $S^3$ forming the space $S^3/\Gamma$. The latter degenerates at $\theta = 0$ fixing the singular $S^1$ in eq. (3.21) with coordinate $\psi$. The Penrose limit (2.2) can now be taken leading to a pp-wave background

$$\begin{aligned} ds^2 &= -4dx^+dx^- - \mu^2(\vec{x}^{\,2} + \vec{y}^{\,2})dx^{+2} + d\vec{x}^{\,2} + dy^2 + y^2 d\tilde{\Omega}_3^2\,, \\ F_{+1234} &= F_{+5678} = 4\mu\,, \end{aligned} \tag{3.22}$$

where $d\tilde{\Omega}_3^2$ is the metric on $S^3/\Gamma$. There exists a singularity at $y = 0$ and the Penrose limit of the $AdS_5 \times S_5/\Gamma$ produces the singular wave fronts $\mathbb{C}^2 \times \mathbb{C}^2/\Gamma$. As orbifolding by $\Gamma$ breaks half the supersymmetries, the Penrose limit of $AdS_5 \times S_5/\Gamma$ also breaks half, leading to 16 surviving supersymmetries. We may blow up the singularity at $y = 0$ by replacing $\mathbb{C}^2/\Gamma$ by an ALE space in such a way that the supergravity equations are still satisfied. Thus, we should look for background with metric and five-form RR field of the form

$$\begin{aligned} ds^2 &= -2dx^+dx^- - S(x^i)dx^{+2} + g_{ij}dx^i dx^i\,, \qquad i,j = 1,\ldots,8\,, \\ F_{+ijk\ell} &= \frac{1}{4!}\epsilon_{ijk\ell mnpq}F_+{}^{mnpq}\,, \end{aligned} \tag{3.23}$$

where $g_{ij}$ is the metric of an 8D-dimensional wave-front $K^8$. The Einstein equations

$$R_{MN} = \frac{1}{96}F_{MKLPQ}F_N{}^{KLPQ}\,, \tag{3.24}$$

reduce to

$$\begin{aligned} \frac{1}{2}\nabla^i\nabla_i S &= \frac{1}{96}F_{+mnpq}F_+{}^{mnpq}\,, \\ R_{ij}(g) &= 0\,. \end{aligned} \tag{3.25}$$

In the Penrose limit of $AdS_5 \times S^5/\Gamma$, $F_{MKLPQ}$ is still given by eq. (2.8) so that

$$ds^2 = -4dx^+dx^- - \mu^2(\vec{x}^{\,2} + S(y))dx^{+2} + d\vec{x}^{\,2} + g_{\mu\nu}dy^\mu dy^\nu \tag{3.26}$$

where $g_{\mu\nu}$ is the metric on the blow-up $\mathbb{C}^2 \times \mathbb{C}^2/\Gamma$ and $S$ satisfies again eq. (3.10).

We may also consider orbifold of the $AdS_5$ space [35, 36]. For example, with $\zeta_0 = X_0 + iX_{11}$, $\zeta_1 = X_1 + iX_2$, $\zeta_2 = x_3 + iX_4$, the AdS hyperboloid (3.17) is written as

$$|\zeta_0|^2 - |\zeta_1|^2 - |\zeta_2|^2 = R^2 \tag{3.27}$$

The $\zeta$'s are expressed as

$$\begin{aligned} \zeta_0 &= R\cosh\rho\, e^{it}\,, \qquad \zeta_1 = R\sinh\rho\, V_1\,, \qquad \zeta_2 = R\sinh\rho\, V_2\,, \\ |V_1|^2 + |V_2|^2 &= 1\,, \end{aligned} \tag{3.28}$$

where $V_{1,2}$ parametrize an $S^3$. We choose a subgroup $\Gamma' \subset \mathrm{SU}(2)$ which acts on $\zeta_1, \zeta_2$. Then $\Gamma'$ acts freely on the $S^3$ of (3.28). The fixed point set of $\Gamma'$ on $AdS_5$ is the singular $S^1$

$$|\zeta_0|^2 = R^2\,, \qquad \zeta_1 = \zeta_2 = 0$$

The Penrose limit can similarly be taken leading to the singular pp-wave background with wave front $\mathbb{C}^2/\Gamma' \times \mathbb{C}^2/\Gamma$

$$\begin{aligned} ds^2 &= -4dx^+dx^- - \mu^2(\vec{x}^{\,2} + \vec{y}^{\,2})dx^{+2} + dx^2 + x^2 d\tilde{\Omega}_3'^2 + dy^2 + y^2 d\tilde{\Omega}_3^2\,, && (3.29)\\ F_{+1234} &= F_{+5678} = \mu\,, && (3.30) \end{aligned}$$

where $\tilde{\Omega}_3^2$, $\tilde{\Omega}_3'^2$ are the metrics on $S^3/\Gamma$ and $S^3/\Gamma'$, respectively. Again, the singularities may be blown up by replacing $\mathbb{C}^2/\Gamma' \times \mathbb{C}^2/\Gamma$ by the two ALE spaces $M^4 \times N^4$. Then, the desingularized Penrose limit of $AdS_5/\mathbb{Z}_{k'} \times S^5/\mathbb{Z}_k$ is

$$ds^2 = -4dx^+dx^- - \mu^2\Big(S(x) + T(y)\Big)dx^{+2} + ds_M^2(x) + ds_N^2(y)\,, \qquad (3.31)$$

where $ds_M^2(x)$, $ds_N^2(y)$ are the metrics on $M^4$, $N^4$, respectively whereas $S, T$ satisfy

$$\nabla^2_{(M)} S = 4\,, \qquad \nabla^2_{(N)} T = 4\,. \qquad (3.32)$$

It is also possible to consider more general orbifolds of the $AdS_5 \times S^5$ geometry. For example consider $(AdS_5 \times S^5)/\mathbb{Z}_k$ defined by the $\mathbb{Z}_k$ action

$$\mathbb{Z}_k: \quad Z_1 \to e^{2i\pi/k} Z_1\,, \qquad Z_1 \to e^{-2i\pi/k} Z_1\,, \qquad \zeta_1 \to e^{2i\pi a/k}\zeta_1\,, \qquad \zeta_2 \to e^{-2i\pi a/k}\zeta_2\,, \qquad (3.33)$$

where $a, k$ relatively prime. Then, in the pp-wave limit, the singular wave fronts $(\mathbb{C}^2 \times \mathbb{C}^2)/\mathbb{Z}_k$ are obtained, where $\mathbb{Z}_k$ as above. It is known that the singularity in this case is terminal [38, 28]. If $a = 1$, $k = 2$ we get 16 supersymmetries ($\mathcal{N} = 8$) whereas if $a = \pm 1\,, k > 2$ there exist only 12 surviving supersymmetries ($\mathcal{N} = 6$). Finally, if $a \neq \pm 1$, we end up with 8 supersymmetries ($\mathcal{N} = 4$). Neither of these singularities have Calabi-Yau resolutions.

The Penrose limits of the $AdS_5 \times S^5$ orbifolds are by now more or less clear. Defining the pp-wave limit of the $AdS_5 \times S^5$ space by the hypersurface in $M^{2,10}$

$$\begin{aligned} X_{10} - X_{11} &= (X_0 + X_9)^2\,, && (3.34)\\ X_{10} + X_{11} &= \frac{\mu^2}{8}\left(X_1^2 + X_2^2 + \cdots + X_8^2\right), && (3.35) \end{aligned}$$

as discussed in the appendix, the orbifolds we have considered above are in fact orbifolds of $\mathbb{C}^4$ with coordinates $Z_1, Z_2, \zeta_1, \zeta_2$ defined above. We have only consider some special cases of such possible orbifolds. However, it is natural to assume that there exists a larger class of $\mathbb{C}^4$ orbifolds (containing the ones we have discussed above) which are wave fronts of Penrose limits of associated $AdS_5 \times S^5$ orbifolds. By desingularizing the $\mathbb{C}^4$ orbifolds, we may obtain smooth pp-wave limits of $AdS_5 \times S^5$ orbifolds, where the locally flat wave fronts are replaced by Ricci-flat ones of appropriate holonomy in order supersymmetry to be preserved.



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### 3.3 $AdS_4 \times S^7$ orbifolds

As in the previous cases, $AdS_4 \times S^7$ can be embedded in $M^{2,11}$ with metric

$$ds_{12}^2 = -dX_0^2 + dX_1^2 + \cdots + dX_{10}^2 + dX_{11}^2 - dX_{12}^2\,, \tag{3.36}$$

as the hypersurface

$$X_0^2 - X_1^2 - X_2^2 - X_3^2 + X_{12}^2 \,=\, R^2\,, \tag{3.37}$$

$$|Z_1|^2 + |Z_2|^2 + |Z_3|^2 + |Z_4|^2 \,=\, 4R^2\,. \tag{3.38}$$

where $Z_1 = X_4 + iX_5,\ Z_2 = X_6 + iX_7,\ Z_3 = X_8 + iX_9,\ Z_4 = X_{10} + iX_{11}$. The parametrization of (3.37)

$$\begin{aligned} X_0 &= R\cosh\rho\,\cos t\,, \qquad X_{12} = R\cosh\rho\,\sin t\,, \\ X_a &= R\sinh\rho\Omega_a\,, \qquad \left(a = 1,\ldots,3, \qquad \sum_a \Omega_a^2 = 1\right), \end{aligned} \tag{3.39}$$

with $0 \le \rho < \infty$ and $0 \le t < 2\pi$ covers the whole of the (3.37) hyperboloid. The angular coordinates $\Omega_a$ parametrize a unit $S^2$ and together with $\rho, t$ are the global coordinates of the $AdS_4$. Similarly, the $S^7$ in eq. (3.38) can be parametrized as

$$\begin{aligned} &Z_1 \,=\, R\cos\theta\, e^{i\psi}\,, \qquad Z_a = R\sin\theta\, U_a\,, \qquad (a = 1,2,3) \\ &|U_1|^2 + |U_2|^2 + |U_3|^2 \,=\, 1\,, \end{aligned} \tag{3.40}$$

where $0 \le \theta < \pi/2$, $0 \le \psi < 2\pi$ and the complex $U_{1,2,3}$ form a unit $S^5$.

We may now consider orbifolds $AdS_4 \times S^7/\Gamma$ where $\Gamma \subset$ SU(4) as in [39]. Such orbifolds acts freely on the $S^5$ of eq. (3.40) but not freely on $AdS_4 \times S^7$ as the hypersurface $\theta = 0$ is the fixed point set of $\Gamma$. In order that supersymmetry is preserved, we should take $\Gamma \subset$ SU(3) $\subset$ SU(4) (1/4 supersymmetry) or $\Gamma \subset$ SU(2) (1/2 supersymmetry). The pp-wave limit of the above orbifolds is the background (2.8) but now the $S^5$ metric $d\Omega_5$ is replaced with the metric on $S^5/\Gamma$. Thus, the wave fronts are not any more $\mathbb{R}^9$ but $\mathbb{R}^3 \times \mathbb{C}^3/\Gamma$. We may replace in this case the singular $\mathbb{C}^3/\Gamma$ with an ALE space of SU(3) holonomy (for $\Gamma \subset$ SU(3)) getting a smooth pp-wave background. The metric is then

$$ds^2 = -2dx^+dx^- - \mu^2\Big((\vec{x}^{\;2} + S(x^i)\Big)dx^{+2} + d\vec{x}^{\;2} + g_{ij}dx^idx^i\,, \qquad i,j = 1,\ldots,6\,, \tag{3.41}$$

where $\vec{x}$ is in $\mathbb{R}^3$, $g_{ij}$ is the metric on the ALE space and $S(x^i)$ satisfies

$$\nabla^i\nabla_i S = 3 \tag{3.42}$$

In the special case where $\Gamma = Z_3 \subset$ SU(3), we may use the ALE space described in the appendix B. In this case, the desingularized pp-wave limit of $AdS_4 \times S^7/\mathbb{Z}_3$ turns out to be

$$\begin{aligned} ds^2 \,=\, &-2dx^+dx^- - \\ &-\mu^2\left(x^2 + \frac{r^2}{4} + \frac{L^2}{24}\ln\left(\frac{L^4 + r^4 - 2L^2r^2}{L^4 + r^4 + L^2r^2}\right) + \right. \\ &\left. + \frac{L^2}{4\sqrt{3}}\left\{\arctan\left(\frac{2r+L}{2\sqrt{3}}\right) - \arctan\left(\frac{2r-L}{2\sqrt{3}}\right)\right\}\right)dx^{+2} + d\vec{x}^{\;2} + d\sigma^2 \end{aligned} \tag{3.43}$$

where $d\sigma^2$ is the metric (B.8). Clearly, at $r \to \infty$ we recover the $\mathbb{R}^3 \times \mathbb{C}^3/\mathbb{Z}_3$ wave fronts.



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### 3.4 $AdS_7 \times S^4$ orbifolds

We may repeat the same procedure as above in the present case as well. We may embed this background in $M^{2,11}$ as before with the role of $AdS_4$ and $S^7$ interchanged. There exist only one orbifold of $S^4$ which preserves supersymmetry. It is the $S^4/\mathbb{Z}_2$ where $\mathbb{Z}_2$ acts on the $S^2$ with metric $d\Omega_2^2$ in eq. (2.12). However, this orbifold is singular at $y = 0$ and it cannot be resolved. Of course, one may consider orbifolds of the $AdS_7$ factor but we will not go into details as this case is similar to the ones already studied above. We should only mention that although $AdS_4 \times S^7$ and $AdS_7 \times S^4$ have the same Penrose limits, their orbifolds do not share the same property.

## 4. pp-waves from $AdS_5 \times S^5$ orientifold

The $AdS_5 \times S^5$ orientifold is the near horizon limit of D3-branes at orientifold planes. The study of D3 branes at orientifolds is similar to orbifolds (see for example [40]). The only difference is the twisted sector which is absent for orientifolds. We will first consider the near horizon of D3-branes on an O3-plane. As the O3-plane breaks the same supersymmetries with the D3, in the near horizon we will have again the maximum 32 unbroken supersymmetries. The near horizon geometry is actually $AdS_5 \times \mathbb{RP}^5$ where $\mathbb{RP}^5 = S^5/\mathbb{Z}_2$. The $\mathbb{Z}_2$ acts by identifying opposite points on the $S^5$ so that there are no fixed points. As a string goes around a non contractible cycle, connecting opposite points in $\mathbb{RP}^5$, it reverses its orientation which is a manifestation of the orientifold projection. There are two types of the orientifold projection. One leads to SO($2N$) $\mathcal{N} = 4$ theory for $N$ D3-branes on the O3-plane, while the other leads to an USp($2N$) theory. These different sting theories are implemented in the $AdS_5 \times \mathbb{RP}^5$ setup by turning on $B_{NS-NS}$ 2-form (discrete torsion) in the non-trivial cohomology class $H^3(\mathbb{RP}^5, \tilde{\mathbb{Z}}) = \mathbb{Z}_2$ [41]. The metric of the $AdS_5 \times \mathbb{RP}^5$ geometry is

$$ds^2 = R^2 \left( d\rho^2 + \sinh^2\rho \, d\Omega_3^2 - \cosh^2\rho \, dt^2 + d\theta^2 + \sin^2\theta \, d\tilde{\Omega}_3^2 + \cos^2\theta \, d\tilde{\psi}^2 \right) \tag{4.1}$$

where $0 \leq \tilde{\psi} < \pi$ and $d\tilde{\Omega}_3^2$ is the metic on $\mathbb{RP}^3 = S^3/\mathbb{Z}_2$. The Penrose limit is then

$$ds^2 = -4dx^+dx^- - \mu^2(\vec{x}^2 + \vec{y}^{\,2})dx^{+2} + d\vec{x}^{\,2} + dy^2 + y^2 d\Omega_3^2 \,, \tag{4.2}$$

where $d\Omega_3^2$ is the metric of $S^3$. Thus, the pp-wave limit is the the same with the $AdS_5 \times S^5$ case.[2]

Next, we will consider the near horizon limit of $N$ D3-branes on an orientifold O7-plane. We will also put D7-branes sitting together with the D3's so that the dilaton is constant and the low-energy theory is conformal [12, 13, 14]. At generic point in the D7-brane moduli space, we have non-conformal field theories living on the D3-branes. There exist 7 different types of singularities which give rise to constant dilaton. These are the Argyres-Douglas points $H_0$, $H_1$ [15, 16] and $H_2$ with $A_0$, $A_1$, and $A_2$ gauge theories on the D7 and $D_4$, $E_6$, $E_7$ and $E_8$ theories resulting from corresponding singularities in F-theory.

[2] We thank the referee for pointing out an error at this point in an earlier version.

Among these singularities, only the $D_4$ can occur for any value of the dilaton. All the others appear at fixed, order one, string coupling. The resulting field theory of coincident $N$ D3 and D7-branes at a $D_4$ singularity is $\mathcal{N}=2$ USp($2N$) SYM theory with a hypermultiplet in the anti-symmetric representation and four hypermultiplets in the fundamental. The $\mathbb{Z}_2$ action has now fixed points on the $S^5$. In fact, the fixed point set (where both the O7 and the D7's are) is $AdS_5\times S^3$. The supergravity description of D3-branes near D7-brane singularities has been described in [43] (for $D_4, E_6, E_7, E_8$) and the metric can be written as

$$\begin{aligned} R^{-2}ds^2 &= d\rho^2+\sinh^2\rho\, d\Omega_3^2-\cosh^2\rho\, dt^2+d\theta^2+\sin^2\theta\, d\psi^2+ \\ &\quad +\cos^2\theta\left(d\omega^2+\sin^2\omega\, d\chi^2+\cos^2\omega\, d\phi^2\right)\,, \end{aligned} \tag{4.3}$$

where $0\leq\theta<\pi/2$ and $\phi$ is periodic with period $2\pi(1-\alpha/2)$. The values of $\alpha$ are $\alpha=1/3$, $1/2$, $2/3$, $1$, $4/3$, $3/2$, $5/3$, for $A_0$, $A_1$, $A_2$, $D_4$, $E_6$, $E_7$ and $E_8$ [42]. The B-fields have generally SL$(2,\mathbb{Z})$ monodromies around the $\phi$-circle.

We may now consider the standard Penrose limit as in eq. (2.2). For this we may first replace $\theta$ with $\theta-\pi/2$ and then take the limit $R\to\infty$ after the transformation eq. (2.7). However, this limit leads to the background in eq. (2.8). Moreover, the monodromies of the B-fields are washed out. However, there exist another limit which preserves the singularities. Defining the coordinates

$$x^+=\frac{t+\psi}{2}\,,\qquad x^-=R^2\left(\frac{t-\psi}{2}\right)\,,\qquad x=R\,\rho\,,\qquad y=R\,\theta\,,\qquad w=R\,\omega \tag{4.4}$$

and taking $R\to\infty$, we end up with the pp-wave limit of (4.3)

$$\begin{aligned} ds^2 &= -4dx^+dx^- -\mu^2(x^2+y^2+w^2)dx^{+2}+dx^2+x^2d\Omega_3^2+ \\ &\quad +dy^2+y^2d\chi^2+dw^2+w^2d\phi^2\,. \end{aligned} \tag{4.5}$$

This can be written as

$$ds^2=-4dx^+dx^- -\mu^2\left(\vec{r}^{\;2}+\frac{|z|^{2-\alpha}}{(1-\alpha/2)^2}\right)dx^{+2}+d\vec{r}^{\;2}+\frac{|dz|^2}{|z|^\alpha}\,. \tag{4.6}$$

where $\vec{r}$ is in $\mathbb{R}^6$. Clearly then, the wave-front geometry is $\mathbb{C}^3\times\mathbb{C}/\mathbb{Z}_n$ where $n=2,3,4,6$ for D3-branes near $D_4, E_6, E_7$ and $E_8$ singularities in F-theory.

## 5. Conclusions

We presented above various orbifolds of $AdS_3\times S^3$, $AdS_5\times S^5$ and $AdS_{4,7}\times S^{7,4}$ as well as orientifolds of $AdS_5\times S^5$. The limiting pp-waves are in general singular. Some of them can be desingularized by replacing their wave fronts (where the singularities are located) by Ricci flat spaces of appropriate holonomy in order to preserve supersymmetry. There exist two ways to view these orbifolds, either as the limiting spaces of $AdS_p\times S^q/\Gamma$ orbifolds, or as orbifolds of the pp-wave limit of $AdS_p \times S^q$. In the latter case, we recall from appendix A that, in general, a D-dimensional pp-wave background can be embedded in $M^{2,D}$ flat space-time as

$$\begin{aligned} X_D - X_{D+1} &= (X_0 + X_{D-1}) \\ X_D + X_{D+1} &= S(X^i), \qquad i = 1, \dots, D-2. \end{aligned} \tag{5.1}$$

We may then consider a discrete action of $\Gamma \subset \mathrm{SO}(D-2)$ on the wave-front coordinates $X^i$. Depending on the geometry of the wave fronts, these orbifolds may or may not preserve supersymmetry. The orbifolds we have considered here are all supersymmetric.

## A. Embedding the pp-waves in higher dimensions

$AdS_5 \times S^5$ which can be embedded in the flat 12D space-time $M^{2,10}$ of $(-,+,+,\dots+,-)$ signature and metric

$$ds_{12}^2 = -dX_0^2 + dX_1^2 + \dots . + dX_{10}^2 - dX_{11}^2 \tag{A.1}$$

Similarly, it is straightforward to show that the pp-wave background

$$ds^2 = -4dx^+dx^- - \mu^2 \vec{r}^{\,2} dx^{+2} + d\vec{r}^{\,2} \tag{A.2}$$

can be embedded in $M^{2,10}$ as the hypersurface

$$X_{10} - X_{11} = (X_0 + X_9)^2 , \tag{A.3}$$

$$X_{10} + X_{11} = \frac{\mu^2}{8} \left(X_1^2 + X_2^2 + \cdots + X_8^2\right) \tag{A.4}$$

Parametrizing the above hypersurface by

$$X_1 = x_1 , \qquad X_2 = x_2 , \qquad \dots \qquad X_8 = x_8 , \tag{A.5}$$

$$X_0 = \mu^2 \left(x_1^2 + x_2^2 + \cdots + x_8^2\right) \frac{x^+}{4} + x^- + x^+ , \tag{A.6}$$

$$X_9 = -\mu^2 \left(x_1^2 + x_2^2 + \cdots + x_8^2\right) \frac{x^+}{4} - x^- + x^+ , \tag{A.7}$$

$$X_{10} = \frac{\mu^2}{16} \left(x_1^2 + x_2^2 + \cdots + x_8^2\right) + 2x^{+2} , \tag{A.8}$$

$$X_{11} = \frac{\mu^2}{16} \left(x_1^2 + x_2^2 + \cdots + x_8^2\right) - 2x^{+2} , \tag{A.9}$$

leads to the pp wave metric (A.2).

More general, the D-dimensional pp-wave metric

$$ds^2 = -4dx^+dx^- - 8S(x^i)dx^{+2} + d\vec{r}^{\,2} , \qquad i = 1, \dots, D-2 \tag{A.10}$$

can be embedded in the (D+2)-dimensional flat space time $M^{2,D}$ with metric

$$ds_{12}^2 = -dX_0^2 + dX_1^2 + \cdots + dX_D^2 - dX_{D+1}^2 , \tag{A.11}$$

as

$$
\begin{aligned}
X_D - X_{D+1} &= (X_0 + X_{D-1})^2 \,, && \text{(A.12)}\\
X_D + X_{D+1} &= S(X^i)\,, && \text{(A.13)}
\end{aligned}
$$

which can be parametrized as

$$
\begin{aligned}
X_1 &= x_1\,, \qquad X_2 = x_2\,, \qquad \dots \qquad X_{D-2} = x_{D-2}\,, && \text{(A.14)}\\
X_0 &= 2Sx^+ + x^- + x^+\,, \qquad X_{D-1} = -2Sx^+ - x^- + x^+\,, && \text{(A.15)}\\
X_D &= \frac{S}{2} + 2x^{+2}\,, \qquad X_{D+1} = \frac{S}{2} - 2x^{+2}\,. && \text{(A.16)}
\end{aligned}
$$

Emmbeding of the pp-wave as intersection of quadratic surfaces in $M^{2,D}$ has been discussed in [5].

## B. A desingularized $\mathbb{C}^3/\mathbb{Z}_3$ space

We will describe the metric of a non-compact 6D space with holonomy SU(3) which is asymptotically $\mathbb{C}^3/\mathbb{Z}_3$. For this, we start by considering the five-dimensional sphere $S^5$, which is the surface

$$
Z_1\bar{Z}_1 + Z_2\bar{Z}_2 + Z_3\bar{Z}_3 = 1\,,
$$

embedded in $\mathbb{C}^3$. By expressing the $Z$'s as [44]

$$
Z_1 = \frac{e^{i\tau}}{(1+r^2)^{1/2}}\,, \qquad Z_2 = \frac{re^{i(\tau+\frac{\chi+\varphi}{2})}}{(1+r^2)^{1/2}}\sin\frac{\theta}{2}\,, \qquad Z_3 = \frac{re^{i(\tau+\frac{\chi-\varphi}{2})}}{(1+r^2)^{1/2}}\cos\frac{\theta}{2}\,, \tag{B.1}
$$

the metric of the $S^5$ turns out to be

$$
ds^2(S^5) = d\Sigma_4^2 + (d\tau + A)^2\,, \tag{B.2}
$$

where

$$
\begin{aligned}
d\Sigma_4^2 &= \frac{dr^2}{(1+r^2)^2} + \frac{1}{4}\frac{r^2}{(1+r^2)}\left(d\theta^2 + \sin^2\theta d\varphi^2\right) + \frac{1}{4}\frac{r^2}{(1+r^2)^2}(d\chi - \cos\theta d\varphi)^2\,, && \text{(B.3)}\\
A &= \frac{r^2}{2(1+r^2)}(d\chi - \cos\theta d\varphi)\,. && \text{(B.4)}
\end{aligned}
$$

The metric $d\Sigma_4$ is just the metric of $\mathbb{CP}^2$ and eq. (B.3) makes manifest the Hopf-fibration of $S^5$, that is a U(1) bundle over $\mathbb{CP}^2$. We may search for SU(3)-holonomy manifolds by making the ansatz

$$
d\sigma^2 = f(r)dr^2 + r^2 d\Sigma_4^2 + r^2 h(r)(d\tau + A)^2\,, \tag{B.5}
$$

where $r$ is a radial coordinate and $f(r), h(r)$ are functions of $r$ which will be determined by demanding the manifold to be Ricci flat. Indeed, the Ricci-flatness condition for this geometry, turns out to be, for $f = 1/h$, the single equation

$$
r^2 hh'' + 3rhh' + r^2 h'^2 - 4h^2 = 0\,. \tag{B.6}
$$

The asymptotically locally flat solution of this equation is

$$h(r) = 1 - \frac{L^6}{r^6} \tag{B.7}$$

and the metric is

$$d\sigma^2 = \frac{dr^2}{1 - L^6/r^6} + r^2 d\Sigma_4^2 + r^2 \left(1 - \frac{L^6}{r^6}\right) (d\tau + A)^2 \,. \tag{B.8}$$

It seems that a singularity exist at $r = L$, which as one suspects can be cured. At $r = L$ we get

$$ds^2 \sim \frac{1}{9} \left(du^2 + 9\, u^2 (d\tau + A)^2\right) + L^2 d\Sigma_4^2 \tag{B.9}$$

and there exist a conical singularity, which is removed if the periodicity of $\tau$ is not $2\pi$ as it seems from (B.1), but rather $2\pi/3$. As a result, although at $r \to \infty$ the space is flat, it is not globally flat in view of the $\tau$ periodicity. In fact it is an ALE space, asymptotically $\mathbb{C}^3/\mathbb{Z}_3$. That it is of SU(3) holonomy can also be proven without much effort.

**Note Added.** During the completion of this work, we became aware of the works [45, 46] and [47] where superstring on $AdS_5 \times S^5/\mathbb{Z}_N$ orbifolds are studied.

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